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ELIMINATION OF RHYTHM FOR PERIODICALLY CORRELATED RANDOM PROCESSES

by

K.S. Voychishin, Ya. P. Dragan

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HUMAN TRANSLATION

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А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*<u>ye</u> initially, after vowels, and after ъ, ь; <u>e</u> elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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ELIMINATION OF RHYTHM FOR PERIODICALLY CORRELATED RANDOM PROCESSES

Voychishin, K.S., Dragan, Ya.P.,

L'vov

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After a Fourier expansion of periodic functions into harmonic components it ceased to be considered as a method of solution of problems of mathematical physics and it was given a physical interpretation in the theory of oscillations, the physical systems began to be treated as sets of oscillators. Then came the time of a certain fetishization of harmonic oscillations, the role of which grew all the more in connection with the successful application of the method of harmonic expansion to the investigation of periodic movements of celestial bodies and with the development of radio engineering.

These circumstances served to a considerable degree as the reason that for the development of methods of analyzing more complex phenomena, such as meteorological, economical, etc. for example, initially the same ideas of expansion of functions into harmonics were used. Thus emerged periodic grammanalysis, the essence of which is from a complex oscillation to identify the simple harmonics, to determine their frequencies, and on the basis of this to judge the presence of specific oscillators in the system being investigated.

However, it was shown that an harmonic expansion has a direct meaning only for linear invariant systems, for which the harmonics are their eigen-functions. Moreover the development of the theory of expansions led to the theory of stationary random processes (SP), which showed the inconsistency of periodic grammanalysis as a method of studying many complex natural phenomena, since in a general case the harmonics here are the result not of the presence of any oscillators, but of the stability of conditions of flow of the process (statistical equilibrium).

As a general model of a "periodic" (rhythmic) phenomenon one should consider [1] periodically correlated random processes (PKSP),

the statistical characteristics of which are periodic. Here rhythm should be considered not as a result of the presence of harmonic oscillators, but as a result of the correlativity of SP harmonics, and the period of rhythmicality treated as a period of this correlativity. From this point of view the harmonic composition of the process indicates only the possible periods of rhythm. For example, if an SP is polyharmonic, then the period of the harmonics has to be a multiple of the period of correlativity, which is confirmed in particular by an analysis of the reactions of systems with periodically changing parameters to a stationary SP. From here it is evident that to find a frequency spectrum for the purpose of exposing the rhythm makes sense only for a polyharmonic SP.

At the present time the statistics of SP are supported only by concepts of stationary processes, therefore its recommendations frequently are inadequate for investigations which are nonstationary, rhythmic phenomena in particular. However, the need arises for reconsidering these recommendations from the point of view of the theory of PKSP as a model of a rhythmic phenomenon. One of such recommendations is the preliminary separation of the high-frequency components of SP. For eliminating the regular periodic trend (cycle) from the random process $\xi(t)$ it is recommended [5] either to select the recordings of its values in the same point of the cycle over the period, or to take for analysis the average variables for the same period. In this case the investigated process represents an additive model of a random rhythmic phenomenon $\xi(t)= f(t)+\nu(t)$, where $f(t)$ - determinate periodic function, and (t) - stationary process.

The purpose of this work is a detailed investigation of the questions brought up above about the model of rhythmic phenomena in the form of PKSP.

Since in the general model in the form of PKSP the rhythm is included in the correlativity of the harmonics, then the elimination of rhythm in this model is equivalent to obtaining a stationary SP from PKSP. In particular it is known [4] that the readings of the values of PKSP after a period of its correlati,ity form a stationary random sequence, i.e., as a result of the read-out the rhythm is eliminated.

Actually, if $\xi(t_0+kT)$ - the sequence of readings ($\overline{k=1, N}$, and t_0 - initial phase), then in view of the periodicity of mathematical expectation of PKSP, i.e., the validity of the correlation $m_\xi(t+T)=m_\xi(t)$ we obtain

$$m_k = E\xi(t_0+kT) = m_\xi(t_0+kT) = m_\xi(t_0). \qquad (1)$$

Since the covariation of PKSP is also periodic, i.e., $b_\xi(t+T, u)=b_\xi(t, u)$, then for the covariation r_{kl}^ξ of the sequence $\{\xi(t_0+kT)\}$ we will have

$$r_{kl}^\xi = E\xi(t_0+kT)\overline{\xi(t_0+lT)} = b_\xi(t_0+kT, [k-l]T) = b_\xi(t_0, [k-l]T). \quad (2)$$

It is evident from formulas (1) and (2) that in the case of a fixed t_0 the random sequence $\{\xi(t_0+kT)$ is a sequence which is stationary in a wide sense, a sequence whose characteristics evidently depend on the initial phase t_0.

Averaging the PKSP values for the period and relating these results to a point with the phase t_0+kTm we obtain the sequence $\eta_k = \frac{1}{T}\int_{t_0+kT}^{t_0+(k+1)T}\xi(s)\,ds$, which can be interpreted as the readings over the period of the moving average with averaging for the period of correlativity; $\eta(t) = \frac{1}{T}\int_t^{t+T}\xi(s)\,ds$, i.e., to obtain $\eta_k=\eta(t_0+kT)$. In this case the characteristics of the second order of such a sequence will be the mathematical expectation $m'_k = m_\eta(t_0=kT)$ and the covariation [6]

$$r_\eta(t+u, t) = R_\eta(u) = \frac{1}{T}\int_{u-T}^{u+T} B_0(v)\,dv. \quad (3)$$

where $B_0(u)$ - null correlation component of PKSP [1].

On the basis of the resulting formulas, taking into account that $m_\eta(t)=m_0$, where m_0 - null coefficient of expansion of the function $m_\eta(\cdot)$ into a Fourier series for mathematical expectation, we find

$$m_k^\eta = m_0. \quad (4)$$

and for covariation -

$$r_{kl}^\eta = R_\eta[(k-l)T] = \frac{1}{T}\int_{(k-l-1)T}^{(k-l+1)T} B_0(u)\,du. \quad (5)$$

In comparing the expressions (1) with (4) and (2) with (5), we arrive at the conclusion that for PKSP the selection of the readings after a period is not equivalent to the selection of the average for the period. For eliminating the dependence of the averages for the period on the initial phase it can be assumed that there is averaging for the period of characteristics (1) and (2) with respect to phase t_0. Then for these averages we obtain

$$M_{t_0}^{T} m_k^{\xi} = m_0 \tag{6}$$

and respectively

$$M_{t_0}^{T} r_{kl}^{\xi} = B_0[(k-l)T], \tag{7}$$

where $M_{t_0}^{T} = \frac{1}{T}\int_0^T dt$ - operator of averaging.

From a comparison of formulas (4) and (6), and also (5) and (7), it is evident that this method is not equivalent to the selection of the averages for the period, since only the mathematical expectations are equal. However, all the rhythm methods are eliminated.

There are specific characteristics in the application of these recommendations (readings over a period or averaging for the period) in the case when the null correlation component $B_0(\cdot)$ is periodic, since its period T_0 should be a multiple to the period of correlativity, i.e., $T_0=nT$ (n - integer [3]). Since formula (4) can be written in the form

$$r_{kl}^{\eta} = \frac{1}{T}\int_0^{(k-l+1)T} B_0(u)\,du - \frac{1}{T}\int_0^{(k-l-1)T} B_0(u)\,du.$$

then, according to the Bolyai-Bohr theorem [2], in the case when the undetermined integral from B_0 is limited it is a periodic function. Therefore the function r_{kl}^{η} as the difference of periodical functions is also periodic.

It follows from the properties of the Fourier expansion that the period of the indeterminate integral of the periodic function is the same as the period of the initial function. Therefore the discrete spectra of functions $B_0(u)$ and $R_{\eta}(u)$ will be concentrated on the same geometric progression of frequencies with the difference $\Lambda_0 = \frac{2\pi}{T} = \frac{\Lambda}{n}$.

Since the readings across the period T will have the result that the spectra of sequences $B_0(kT)$ and $R_\eta(kT)$ will be concentrated on the set of frequencies $\{k\Lambda_0, \overline{k=0, n-1}\}$, then it is evident from here that in this case the averages by phases and the averages for a period will produce the same set of frequencies, but their energies are different.

For an illustration of this from formula (3) we find that if

$$B_0(u) = \sum_{k=-\infty}^{\infty} b_m e^{im\lambda_0 u}. \quad (8)$$

then

$$R_\eta(u) = 2 \sum_{n=-\infty}^{\infty} b_m \frac{\sin m\Lambda_0 T}{m\Lambda_0 T} e^{im\Lambda_0 u}.$$

Then, taking $\Lambda_0 T = \frac{2\pi}{n}$ into account, we obtain

$$r^\eta_{kl} = 2 \sum_{k=-\infty}^{\infty} b_m \frac{\sin m\frac{2\pi}{n}}{m\frac{2\pi}{n}} e^{im\frac{2\pi}{n}(k-l)}.$$

Keeping in mind that $e^{iq2\pi}=1$ in the case of a whole q, we see that in the last expression a "sticking" of the coefficients of expansion takes place, and then, designating

$$B_l = 2 \sum_{q=-\infty}^{\infty} b_{nq+l} \frac{\sin (nq+l)\frac{2\pi}{n}}{(nq+l)\frac{2\pi}{n}}.$$

we obtain

$$r^\eta_{kl} = \sum_{l=0}^{n-1} B_l e^{il\frac{2\pi}{n}(k-l)}. \quad (9)$$

For the covariation of the averaged sequence on the basis of formula (7) we find

$$M^T r^l_{kl} = \sum_{m=-\infty}^{\infty} b_m e^{im\frac{2\pi}{n}(k-l)}$$

or, having designated $B'_l = \sum_{q=-\infty}^{\infty} b_{nq+l}$, we convert this expression to the form

$$M^T r^{a}_{kl} = \sum_{l=0}^{n-1} B'_l e^{il\frac{2\pi}{n}(k-l)}. \tag{10}$$

A comparison of formulas (9) and (10) confirms what was said above. The meaning of the last correlations becomes understandable when it is taken into account that if the null correlation component is periodic with a period $T_0 = nT$, i.e., it can be presented in the form

$$\xi(t) = \sum_{k=-\infty}^{\infty} \zeta_k e^{ik\Lambda_0 t}.$$

then the PKSP will be polyharmonic. The correlation function of such a process

$$b(t, u) = \sum_{k,l=-\infty}^{\infty} E\zeta_k \bar{\zeta}_l e^{i(k-l)\Lambda_0 t} e^{ik\Lambda_0 u}$$

or after simple conversions

$$b(t, u) = \sum_{l=-\infty}^{\infty} e^{il\Lambda_0 t} B_l(u),$$

where $B_l(u) = \sum_{k=-\infty}^{\infty} r_{k,k-l} e^{ik\Lambda_0 u}$.

Since all $B_j(\cdot)$ have the same period T_0, as $B_0(\cdot)$, then $b(t, \cdot)$ as the sum of the periodic functions possesses the same period and can be presented in the form

$$b(t_0, u) = \sum_{k=-\infty}^{\infty} e^{ik\Lambda_0 u} C_k(t_0).$$

where functions $C_k(\cdot)$ are periodic with a period T_0. Therefore the covariation of readings across the period

$$b[t_0, (k-l)T] = \sum_{l=0}^{n-1} e^{il\frac{2\pi}{n}(k-l)} C_l(t_0).$$

where $C_l(t_0) = \sum_{q=-\infty}^{\infty} C_{nq+l}(t_0)$.

Finally, for averaging by phase t_0 we obtain

$$B[(k-l)T] = \sum_{l=0}^{n-1} e^{ik\frac{2\pi}{T}(k-l)} C_l,$$

where $C_l = \frac{1}{T}\int_0^T C_l(t_0)\,dt_0.$

From here it is evident that with any of the methods mentioned above for working polyharmonic PKSP we will obtain a stationary approximation, the spectrum of which is concentrated on the set $\{k\Lambda_0,\ k=0,\ \overline{n-1}\}$. Then as a result of the correlation $T_0=nT$ in the given case it is possible to estimate the period of correlativity.

If the PKSP is not polyharmonic, then these methods, as already mentioned above, give a stationary approximation which does not contain information about the rhythm of the initial process.

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